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FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS

NATIONAL DIPLOMA IN ENGINEERING:
ANALYTICAL CHEMISTRY

MODULE MAT2AW2
ENGINEERING MATHEMATICS 2

CAMPUS DFC

JUNE 2014EXAMINATION

DATE 02/06/2014 **SESSION 12H30**

ASSESSORS MR P SELOANE

INTERNAL MODERATOR **MRIK LETLHAGE**

DURATION 3 HOURS **MARKS 100**

SURNAME AND INITIALS:_____

STUDENT NUMBER:_____

COURSE:_____

LECTURER: _____

CONTACT NO: _____

NUMBER OF PAGES: 19(VERIFY CORRECT NUMBER OF PAGES IN YOUR SCRIPT))
INSTRUCTIONS : ANSWER ALL THE QUESTIONS
USE THE BLANK PAGES AT THE BACK TO DO ROUGH WORK
NO PAGES SHOULD BE REMOVED FROM THIS PAPER.
USE ONLY BLUE OR BLACK INK TO WRITE. NO PENCIL.
USE PAGE 19 TO REDO ANY QUESTION YOU HAVE CANCELLED.

REQUIREMENTS :INFORMATION BOOKLET
: NON-PROGRAMMABLE SCIENTIFIC CALCULATOR

INSTRUCTIONS

SHOW ALL THE STEPS TAKEN AND GIVE YOUR FINAL ANSWERS CORRECT TO TWO DECIMAL PLACES, WHERE APPLICABLE. USE THE BLANK PAGES FOR ROUGH WORK. ANYTHING WRITTEN IN PENCIL WILL NOT BE MARKED.

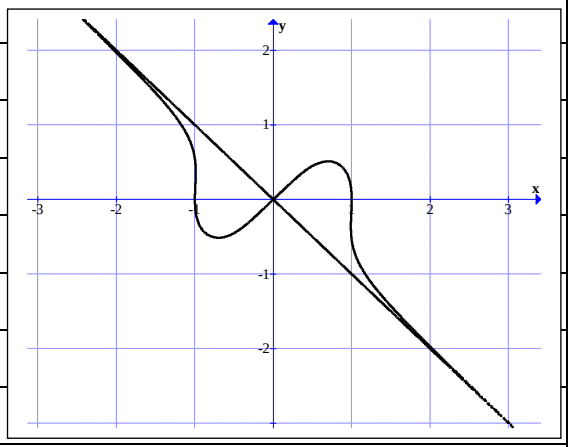
QUESTION 1

1.1 Find the second derivative of $y = \sqrt{x}\sin\sqrt{x}$ (4)

1.2 Given $y = \dfrac{e^{x^2}\tan^{-1}x}{\sqrt[4]{2x^2+3}}$, evaluate $\dfrac{dy}{dx}$ and write the answer in its simplest form. (4)

1.3Let $x = 3\cos 3t$ and $y = 2\sin^2 3t - 1$. Find $\frac{d^2y}{dx^2}$ (5)

1.4 The curve defined by the equation $(x^3 + y^2)^2 = x^2 - y^2$ is called a lemniscuses. Find the first derivative. (5)



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QUESTION 2

2.1 Let $z = x\cos 2y$. Find $\frac{\partial z}{\partial y}$, $\frac{\partial z}{\partial x}$ and $\frac{\partial^2 z}{\partial x \partial y}$ in their simplest forms. (4)

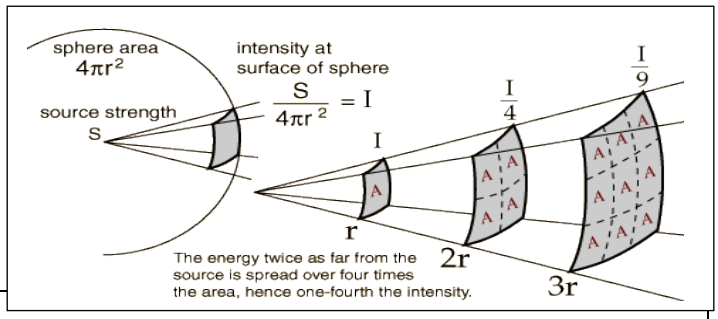
2.2 The intensity of a beam of electromagnetic radiation is the energy it delivers per second. It is given by the formula $I = \frac{S}{4\pi r^2}$. Use partial derivatives to determine the percentage change in intensity I if the power, S, increases by 2% and the distance r, increases by 5%. Is the power increasing or decreasing? (5)

I = intensity [w/m^2]

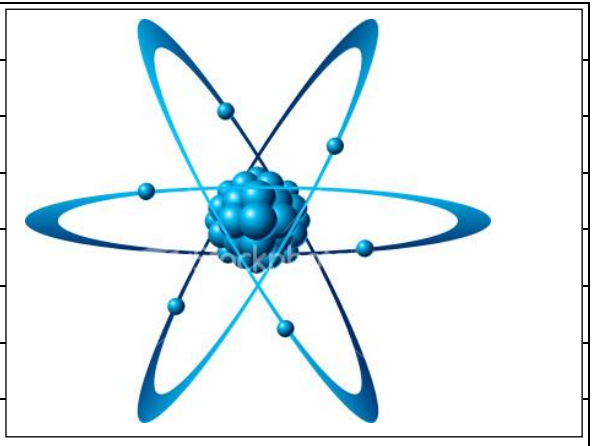
S = power of source [watts]

r = distance [m]

$4\pi r^2$ = surface area of sphere.



2.3 The pressure (P), volume (V) and temperature (T) of a mole of ideal gas are related by the equation $PV = 8.31T$, where *P* is measured in kilopascals, *V* incubiclitres and *T* in Kelvin. If amole of gas is at a temperature (T) of 273 Kelvin and the volume (V) changes from $12l^3$ to $12.3l^3$,use partial derivatives to find the approximate change in the temperature (T) if the pressure (P) must remain constant. Is the temperature increasing or decreasing? (5)



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QUESTION 3

Evaluate the following integrals. Show all the integration steps.

3.1 $\int \frac{-4}{\sqrt{6x^5}}\,dx$ (2)

3.2 $\int_0^{\frac{\pi}{4}} \frac{\sin x}{\sqrt{\cos^3 x}}\,dx$ (3)

3.3 $\int \frac{\cosh 3x}{e^{2x}}\,dx$ (3)

3.4 $\int \frac{x+3}{2x^2-3x-2}dx$ (5)

3.5 $\int \ell n|x+1|\,dx$ (5)

3.6 $\int \sqrt{x}\tan^{-1}\sqrt{x}\,dx$ (5)

3.7 $\int (x+2)\sqrt{3x-x^2-1}\,dx$ (5)

3.8 $\int_0^1 \frac{x^2}{\sqrt{4-x^2}}\,dx$ (8)

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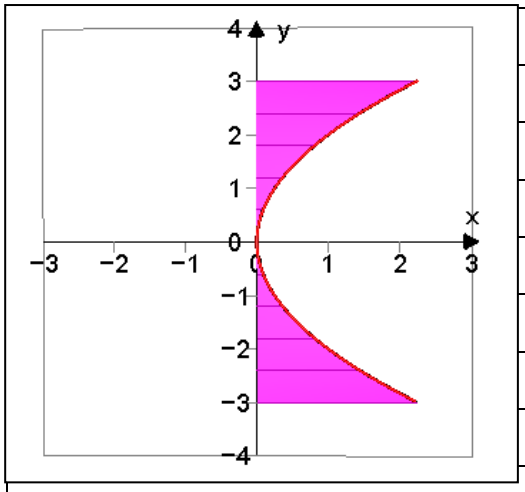
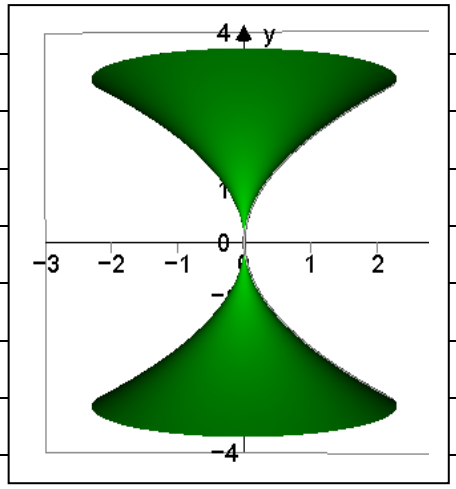
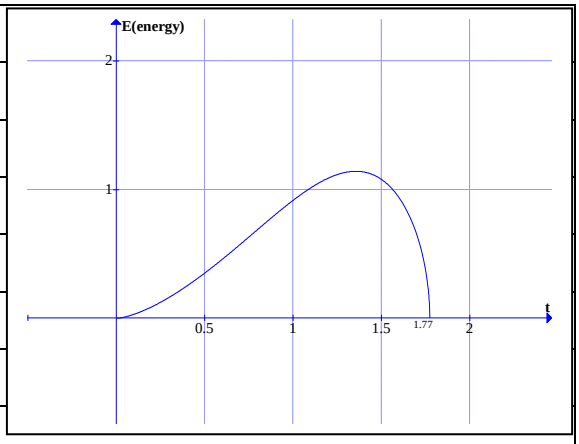
QUESTION 4

4.1 Find the volume of the solid generated by revolving the area bounded by the parabola $y = 2\sqrt{x}$ and the y-axis about the y-axis between the limits $-3 \le y \le 3$. (5)





4.2 Find the r.m.s. value of the energy released by a heat wave spike defined by the function $E = \sqrt{t \sin t^2}\, dt$ between $t = 0$ and $t = 1.77$ (5)



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QUESTION 5

5.1 Test the following DE for exactness and solve it using any method.

$$\left(x+e^{y}\right)dy=\left(e^{x}-y\right)dx \qquad (4)$$

5.2 Solve the following initial value problem

$xy\dfrac{dy}{dx} = 2x^2 + y^2$ given that $y(1) = 2$. (6)

5.3 Show that the general solution of the DE

$$\frac{dy}{dx} - 2y\tan x = y^2\tan^2 x \text{ is } y = \frac{3\sec^2 x}{3C - \tan^3 x} \quad (8)$$

5.4 A copper wire at $98^{\circ}C$ is placed in running water at $40^{\circ}C$. After 4 minutes the temperature (T) of the wire was $58^{\circ}C$. Estimate, showing your calculations the temperature (T) of the wire after 7.3 minutes if the wire obeys Newton's law of cooling. $\frac{dT}{dt} = -k(T - T_a)$, where T represents the temperature of the object, T_a the ambient temperature and t the time. (4)

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TOTAL MARKS: 100

RE-DO ANY QUESTION YOU MAY HAVE CANCELLED HERE!